COST OPTIMIZATION OF MULTISTAGE ROCKETS

By Paul R. Hill

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The optimization of large rocket systems for minimum cost rather than minimum system weight may possibly save millions of dollars per flight. A simple method of optimizing tandem-staged rocket systems for either minimum weight or minimum cost is reviewed. The easily used recursion equations presented apply equally well to either objective. The method applies to any number of stages and takes into account the parameters of stage jet velocity, stage inert mass fraction, and stage specific cost or dollars per pound. It is shown that when the unit cost of the several tandem stages are equal a minimum weight solution is a minimum cost solution. When the unit cost varies markedly from stage to stage it is important to select the relative stage weights to minimize the cost. An example of a rocket system to carry a 20,000 lb payload to escape speed utilizing a solid fuel first stage with hydrogen fueled second and third stages showed the minimum cost system weighs 21 percent more but costs 25 percent less than the corresponding minimum weight system. No mathematical optimization method can be expected to accurately predict final system weights and costs, but can be very worthwhile in quickly orienting the detailed and laborious hardware design studies which must eventually be made.

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Staged rocket systems have been utilized in the United States since the late 1940's. All early stage optimization procedures were based on minimizing the total rocket system weight which would achieve the required payload weight and velocity as described in references 1 through 3. As the rockets became larger and more costly, and as the freedom of choice between different types of rockets increased, it became increasingly important to optimize with respect to cost rather than with respect to total weight. Such a cost optimization procedure would be expected to indicate a low unit cost rocket for the first stage because most of the weight of the entire system is in the first stage. It should also indicate the use of more exotic high performance rockets for the smaller upper stages. The method must give, in a straightforward manner, the correct relative weight and cost of each stage to minimize the total cost. In large rocket systems the savings from optimizing with respect to cost rather than weight can be expected to run into millions of dollars per flight.

The first mathematical method suitable for cost optimization was published by C. H. Builder in 1958, ref. 4. A second method quite similar to that of Builder, but easier to apply, was published by L. Shenfil and R. F. Tangren in 1960, ref. 5. A third mathematical method of cost optimization was published by the present author in 1963, ref. 6.

Surprisingly, the addition of the parameter of rocket unit cost needed for cost optimization to the others of jet velocity and stage inert mass fraction as also needed for minimizing weight, has not necessarily increased the complexity of the analysis. That the amount of work involved in utilizing such methods is not large, even for a desk computer, appears to be not generally appreciated. This paper will attempt to make this point clear by example, following the method of ref. 6, and to also illustrate the type of circumstance where cost minimization is definitely superior to weight minimization. Derivations are given in reference 6 and are not repeated here.

Let us first define the symbols:

c stage specific cost, dollars/lb

C_R total rocket cost per firing, dollars

m_j ratio of the loaded mass of the j^{th} stage to the total mass (including payload) carried above the j^{th} stage,

$$\frac{W_j}{W_{j+1} + W_{j+2} + \cdots + W_n + W_L}$$

r_j j^{th} stage mass ratio, the ratio of total systems mass at the beginning of the j^{th} stage firing to the total system mass at the end of the j^{th} stage firing $= \dfrac{W_j + W_{j+1} + \cdots + W_n + W_L}{\beta W_j + W_{j+1} + \cdots + W_n + W_L}$

This is the mass ratio used in the speed equation.

v_j jet velocity, fps

V_i ideal velocity at the n^{th} stage burnout. It is the velocity that would be obtained if there were no drag or gravity losses.

W_j stage weight, including the interstage structural and control weights that remain fixed to the stage at separation

W_g gross weight at launch, lb

W_L payload weight, lb

β_j j^{th} stage inert weight fraction, the ratio of the stage weight empty to the weight loaded with propellants

Subscripts: $1, 2, \cdots j, \cdots n = 1^{st}, 2^{nd}, \cdots j^{th}, \cdots n^{th}$ stages.

The problem is defined as follows:

Given the values of stage specific cost, jet velocity, inert weight fraction, payload weight, and required velocity, find the values of rocket stage weights W_j for minimum cost.

The optimization procedure is presented in Fig. 1. Exactly as with most procedures that optimize for minimum weight, the mass ratio for the first stage r_1 must be adequate for the desired vehicle performance. First assume a mass ratio r_1. The other stage mass ratios are calculated from eq. 1 and the ideal velocity from eq. 2. Iterate r_1 until V_i assumes the desired value and the optimization is complete. Note that mass ratios computed during the iteration are all minimum cost stage mass ratios for the corresponding value of V_i given by eq. 2. The equations in Fig. 1 will handle 3 stages. If there are 4 compute r_4 from:

$$r_4\beta_4 = 1 - \frac{v_1/v_4}{m_1m_2m_3}\left[\frac{c_4}{c_1}\left(1 - \beta_1r_1\right) + r_1 - 1 + \frac{v_2}{v_1}m_1\left(r_2 - 1\right) + \frac{v_3}{v_1}m_1m_2\left(r_3 - 1\right)\right] \qquad 1(c)$$

where $m_3 = r_3\left(1 - \beta_3\right)\Big/\left(1 - \beta_3r_3\right)$

Formulas for more stages can be derived by inspection.

As first pointed out in ref. 1 the unit costs appear as ratios. Hence the analysis is also valid if only the relative costs are known and not the absolute values. It is also clear that if there is no difference in the relative unit cost of the stages, equations 1 and 2 reduce to an economical method of optimizing a rocket system for minimum weight. Actually, in this case the minimum weight system is the minimum cost system. The method, then, is in general perfectly suitable for both types of staging optimization and results in the economy of a single computer method in place of two.

Fig. 2 gives the equations to obtain stage weights and cost. Eq. 3 gives the stage weights starting with the n^{th} or top stage, utilizing the values of m already computed. Equations 1 and 3 are recursion equations wherein succeeding equations make use of a value found in the preceding equation. Equations 4 and 5 are the obvious ones for gross weight and total rocket system cost.

Examples: Let us compare a rocket system optimized for minimum cost with a system optimized for minimum weight. Let it be required to launch a 20,000 lb payload to escape speed with a 3-stage rocket system. From previous trajectory experience it is known that an ideal velocity of 40,000 ft/sec can accomplish this. First consider a JP4-LOX first stage with H_2-LOX upper stages; then a solid fuel first stage with H_2-LOX upper stages.

Approximate physical properties assumed for developed rocket motors are given in Fig. 3. These costs do not include handling and launching costs. Using equations 1 and 2, four or five iterations in r_1 will yield the r's and V_i to 4 or 5 significant figures.

Fig. 4 gives the resulting stage weights and costs for the JP4-LOX first stage. The minimum cost system weighs 6.5 percent more and costs 8 percent less than the minimum weight system. Fig. 5 gives equivalent results for the solid first stage. The minimum cost system weights 21 percent more and costs 25 percent less than the minimum weight system.

Comparing the two cases it is more important to optimize with respect to cost the systems with the largest variation in unit costs. This is consistent with the former observation that where there is no variation in unit costs there is no difference in optimization for minimum weight or minimum cost.

Another comparison must be made. The least cost optimization with the JP4-LOX first stage costs more than the other system, whichever way optimized. Hence the largest gains may be made by the selection of the proper type of system components. The main advantage of a good cost optimization system probably lies in its ability to make the proper comparison between different systems. From this viewpoint the ability of the method to indicate a preliminary design through the optimum proportioning of the various stages is a bonus.

There is considerable latitude for freedom of choice as to what costs should be included in arriving at reasonable unit costs. Perhaps the most difficult concept to introduce in arriving at unit cost is the cost of development. These costs could, in some cases, be so large as to overshadow all others. The usual procedure is to shun development as much as possible, but to amortize expected development costs over the total anticipated number of firings. However, if only developed rockets are to be used and the unit cost is selected accordingly, then such motors can only be selected as near as practicable to the ideal sizes indicated by this analysis.

In comparing different types of systems the system reliability, if low, can enter strongly into the cost comparison. As observed in reference 6 the reliability of the various stages does not affect the relative size of the various stages and does not enter the optimization procedure. It does enter the comparison of an optimum system of one reliability with another optimum system having a different reliability. A useful concept for this comparison is the cost per successful firing. This is obtained by dividing the total cost per system by the system reliability, which in turn is just the product of the various stage reliabilities. For further detail see ref. 6. If reliabilities are high they do not have a strong influence on the choices to be made. For this reason, and because of the low confidence in reliability numbers for large rockets, the subject is not pursued further here.

One last point must be made. No mathematical optimization procedure for staged rockets can be expected to predict precise values for the system weights or costs, and such methods must not be expected to replace detailed design studies. However, optimization procedures can be expected to aid in the selection of the best combination of rockets for the booster system and to quickly orient design studies. The very small effort involved in making a few optimization calculations is almost nothing compared to their value in selecting systems and orienting the detailed and laborious design studies which must be done eventually on the system selected.

References

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Figure 1. THE OPTIMIZING PROCEDURE

Assume a value of r_1 and calculate:

$$r_2\beta_2 = 1 - \frac{v_1/v_2}{m_1}\left[\frac{c_2}{c_1}\left(1 - \beta_1 r_1\right) + r_1 - 1\right] \qquad 1(a)$$

where $m_1 = r_1(1 - \beta_1)\big/(1 - \beta_1 r_1)$

$$r_3\beta_3 = 1 - \frac{v_1/v_3}{m_1 m_2}\left[\frac{c_3}{c_1}\left(1 - \beta_1 r_1\right) + r_1 - 1 + \frac{v_2}{v_1} m_1\left(r_2 - r_1\right)\right] \qquad 1(b)$$

where $m_2 = r_2(1 - \beta_2)\big/(1 - \beta_2 r_2)$

$$V_i = v_1 \ln r_1 + v_2 \ln r_2 + \cdots + v_n \ln r_3 \qquad 2$$

Iterate r_1 until V_i attains desired value.

Figure 2. WEIGHTS AND COSTS

$$W_n = \left(m_n - 1\right) W_L$$

$$W_{n-1} = m_n\left(m_{n-1} - 1\right) W_L \qquad 3$$

$$W_{n-2} = m_n m_{n-1}\left(m_{n-2} - 1\right) W_L \quad \text{etc.}$$

$$W_G = W_1 + W_2 + \cdots + W_n + W_L \qquad 4$$

$$C_R = c_1 w_1 + c_2 w_2 + \cdots + c_n w_n \qquad 5$$

Figure 3. ASSUMED ROCKET PROPERTIES

Quantity	Stage-1	Alt. Stage-1	Stage-2	Stage-3
Propellants	JP4-LOX	Solid	H_2-LOX	H_2-LOX
Stage specific cost, $/lb	10	5	20	20
Stage mass fraction	0.13	0.13	0.14	0.14
Jet velocity, ft/sec	8500	8200	13,500	13,500

Figure 4. PROBLEM 1-a LAUNCH 20,000 lb to V_L = 40,000 ft/sec

JP4-LOX FIRST STAGE

Quantity	Stage 1	Stage 2	Stage 3	Total
Min.-wt. stage wt., lb	845,900	720,100	110,400	1,676,400
Min.-cost " " "	1,269,700	431,800	83,500	1,785,000
Min. wt. cost				$25,069,000
Min. cost cost				$23,003,000

Figure 5. PROBLEM 1-b SAME AS 1-a EXCEPT SOLID FIRST STAGE

Quantity	Stage 1	Stage 2	Stage 3	Total
Min.-wt. stage wt., lb	812,900	790,600	116,100	1,719,600
Min.-cost " " "	1,673,800	340,300	73,100	2,087,200
Min. wt. cost				$22,200,000
Min. cost cost				$16,640,000

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